Programming Paradigms
First session about logic programming

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This session concerns the `Datalog` language; for implementation, use the DrRacket version of Datalog. Some of the problems below involve programming, others do not.

Problem 1

The goal of this problem is to understand and modify the following Datalog program.

    father(C,D) :− parent(C,D), male(D).
    mother(C,M) :− parent(C,M), female(M).

1. Describe, in your own words, what the program computes.

   This program computes the father relation and the mother relation.

2. Add the family facts about the royal family from the podcast to the program and extend the program with rules that define *sisters* and *aunts*. You will also have supply facts about gender (all members of the royal family are male or female). Is there anything unexpected? How would you modify your program to deal with this?

   Here is a Datalog program:

   parent(christian,mary).
   parent(isabella,mary).
   parent(vincent,mary).
   parent(josephine,mary).
   parent(henrik,marie).
   parent(athena,marie).

   female(mary).
   female(marie).
   female(isabella).
   female(josephine).
   female(athena).

   sister(X,Y) :- female(X), female(Y), parent(X,Z), parent(Y,Z).

   aunt(X,Y) :- sister(Y,Z), parent(X,Z).

   Unfortunately, this program tells you that you are your own sister, as the clause defining the sister predicate does not require X and Y to be different. Moreover, a consequence is that your female parent also becomes your aunt.

The solution is to amend the clause for the sister predicate requiring X and Y to be different. We also need to define a predicate for this.

different(mary,marie).
different(marie,mary).
different(isabella,josephine).
different(josephine,isabella).

sister(X,Y) :- female(X), female(Y), parent(X,Z), parent(Y,Z), different(X,Y).

Problem 2

What is the difference between the following two Datalog programs?

    happy(Person) :− rich(Person),famous(Person).

and

    happy(Person) :− rich(Person).
    happy(Person) :− famous(Person).

Find a collection of facts that you can add to both programs and a query that shows the difference.

The different is that first program tells us that a person is happy if she is rich and famous; comma represents a conjunction (an AND), whereas the second program specifies that a person is happy if she is rich or famous; the use of two clauses specifies a disjunction (an OR).

The following version with facts included illustrates this.

happy1(Person) :- rich(Person),famous(Person).

happy2(Person) :- rich(Person).
happy2(Person) :- famous(Person).

rich(billgates).
rich(jeffbezos).
rich(vicechancellor).

famous(billgates).
famous(jeffbezos).

The query happy1(vicechancellor) fails, but the query happy2(vicechancellor) succeeds.

Problem 3

The goal of this problem is to understand the abstract syntax of Datalog. Identify the syntactic categories that appear in the Datalog program

```
rich(beyonce).
famous(beyonce).
happy(Person):− rich(Person),famous(Person).
```

The syntactic categories are the ones found in the formation rules for Datalog presented in the podcast for this session.

rich and famous and happy are predicates.

rich(beyonce) and famous(beyonce) are atoms. They are even ground atoms. They appear in the program as clauses (more specifically as facts).

beyonce is a term.

Person is a variable.

happy(Person):− rich(Person),famous(Person). is a clause.

Problem 4

Here is map of South America.

1. Write a Datalog program that specifies the following two conditions

   - Red, green and blue are distinct colours. Use a 2-place predicate `distinct` for this.

   - A 4-place predicate `colouring` that specifies that the colours of Colombia, Ecuador, Brazil and Peru should be different.

```prolog
distinct(red,green).
distinct(red,blue).
distinct(green,blue).
distinct(green,red).
distinct(blue,red).
distinct(blue,green).

three_colouring(COL,BRA,ECU,PER) :-

distinct(COL,BRA),
distinct(COL,ECU),
distinct(COL,PER),
distinct(ECU,PER),
distinct(PER,BRA).
```

2. Write the Datalog program that find a colouring of the section of the map that is restricted to Colombia, Ecuador, Brazil and Peru.

<div style="border:1px solid">

We perform the Datalog query

colouring(C,E,B,P)?

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Two problems related to the miniproject

Problem 5

The goal of this problem is to write and test a Datalog program that will compute the transitive closure of a relation.

1. Given a *directed* graph, write a Datalog program to compute the all-paths relation, i.e. the transitive closure of the graph.

2. Extend the program such that it can compute all vertices that appear on a path from a vertex x to a vertex y.

Problem 6

The goal of this problem is to write and test Datalog programs that compute relations that satisfy a certain property.

1. Given a *directed acyclic* graph where each edge is either blue or red, write a Datalog program to compute all paths where every edge in the path has the same colour.

2. Given a directed acyclic graph, write a Datalog program to compute all paths of even length.