Programming Paradigms
Second session about logic programming

Hans Hüttel

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Most of the problems today are not programming problems but have to do with the learning goal of understanding the underlying theory.

Problem 1

Given the Datalog program:

```
loves(rose, jack).
loves(jack, rose).
loves(caledon, rose).
happy(X) :- loves(X, Y), loves(Y, X).
```

What are all the possible interpretations?

Solution:
The Herbrand universe U_P is

$$U_P = \{\texttt{rose}, \texttt{jack}, \texttt{caledon}\}$$

The Herbrand base B_P is

$$B_P = \{\texttt{loves}(t_1, t_2) \mid t_1, t_2 \in U_P\} \cup \{\texttt{happy}(t) \mid t \in U_P\}$$

The set of possible interpretations is the family of subsets

$$I_P = \{S \mid S \subseteq B_P\}$$

Problem 2

Given the Datalog program above with the interpretation:

$$I = \Big\{\texttt{loves}(\texttt{rose}, \texttt{jack}), \quad \texttt{loves}(\texttt{caledon}, \texttt{caledon}), \quad \texttt{happy}(\texttt{rose})\Big\}$$

which of the following ground atoms and rules are true under the interpretation:

- $\texttt{loves}(\texttt{rose}, \texttt{jack})$?
- $\texttt{loves}(\texttt{jack}, \texttt{rose})$?
- $\texttt{happy}(\texttt{rose})$?
- $\texttt{happy}(\texttt{jack})$?
- $\texttt{happy}(\texttt{caledon})$?
- $\texttt{happy}(\texttt{caledon}) \Leftarrow \texttt{loves}(\texttt{caledon}, \texttt{caledon}), \texttt{loves}(\texttt{caledon}, \texttt{caledon})$?
- $\texttt{happy}(\texttt{rose}) \Leftarrow \texttt{loves}(\texttt{rose}, \texttt{jack}), \texttt{loves}(\texttt{jack}, \texttt{rose})$?

Solution: Facts mentioned in the program but not in the interpretation are not true.

- $\texttt{loves}(\texttt{rose}, \texttt{jack})$ is true, since $\texttt{loves}(\texttt{rose}, \texttt{jack}) \in I$
- $\texttt{loves}(\texttt{jack}, \texttt{rose})$ is false since $\texttt{loves}(\texttt{jack}, \texttt{rose}) \notin I$

- happy(rose) is true since happy(rose) $\in I$
- happy(jack) is false since happy(jack) $\notin I$
- happy(caledon) is false since happy(caledon) $\notin I$
- happy(caledon) $\Leftarrow$ loves(caledon, caledon), loves(caledon, caledon) is true, since this is an instance of the rule where the premises are true (since loves(caledon, caledon) $\in I$ and loves(caledon, caledon) $\in I$) and the conclusion happy(caledon) also is.
- happy(rose) $\Leftarrow$ loves(rose, jack), loves(jack, rose) is true, since happy(rose) is true.

Problem 3

Given the Datalog program above which of these interpretations are *models*?

$$I_1 = \Big\{\texttt{loves(rose, jack)}\Big\}$$

$$I_2 = \Big\{\texttt{loves(rose, jack), loves(jack, rose), loves(caledon, rose)}\Big\}$$

$$I_3 = \Big\{\texttt{loves(rose, jack), loves(jack, rose), loves(caledon, rose),}$$
$$\texttt{happy(rose)}\Big\}$$

$$I_4 = \Big\{\texttt{loves(rose, jack), loves(jack, rose), loves(caledon, rose),}$$
$$\texttt{happy(rose), happy(jack)}\Big\}$$

$$I_5 = \Big\{\texttt{loves(rose, jack), loves(jack, rose), loves(caledon, rose),}$$
$$\texttt{happy(rose), happy(jack), happy(caledon)}\Big\}$$

Which model is *minimal*?

Solution: Again, remember that facts mentioned in the program but not in the interpretation are not true.

I_4 and I_5 are models. I_4 is minimal.

Problem 4

Given the Datalog program

```
god(odin).
son(odin,thor).
son(odin,baldr).
son(thor,mothi).
son(thor,magni).
demigod(X) :- son(Y,X), god(Y).
mortal(X) :- son(Y,X), demigod(Y).
```

Compute the *minimal model* using the immediate consequence operator T_p.

Show the facts inferred in each iteration.

Solution:

$T_p(\emptyset) = \emptyset$

$T_p^1(\emptyset) = T_p^0(\emptyset) \cup \{\texttt{god(odin), son(odin, thor), son(odin, baldr), son(thor, mothi), son(thor, magni)}\}$

$T_p^2(\emptyset) = T_p^1(\emptyset) \cup \{\texttt{demigod(thor), demigod(baldr)}\}$

$T_p^3(\emptyset) = T_p^2(\emptyset) \cup \{\texttt{mortal(mothi), mortal(magni)}\}$

Problem 5

Determine if the Datalog program:

```
A(X) :- B(X, X).
C(X,X) :- D(X).
A(X) :- B(X,Y), not C(X,Y).
B(X,Y) :- C(X,Y), not D(X).
```

is stratified. If so, compute its stratification.

Solution: We build the precedence graph. Its edges are

$$B \to^{+} A$$
$$D \to^{+} C$$
$$C \to^{-} A$$
$$B \to^{+} A$$
$$C \to^{+} B$$
$$D \to^{-} B$$

It does not contain any cycles with negative edges. Moreover, every variable that occurs in a negative atom also occurs in a positive atom in the body.

From the graph, we see that the stratification is

$$P_0 = \{D\}, P_1 = \{C\}, P_3 = \{B\} = P_4 = \{A\}$$

A problem related to the miniproject

Assume we can travel from city to city by railway lines of types A, B or C.

Write Datalog programs to answer the questions:

- Find all routes from x to y by any means.
- Find all routes from x to y by railway lines of type A only.
- Find all routes from x to y by lines of type A, but only if there is no route from x to y by lines of other types.